INDIAN INSTITUTE OF TECHNOLOGY

PHYSICS TUTORIAL 8

1 Which of the following wave functions cannot be solutions of Schrödinger's equation for all values of x? Why not? (a) $\psi = A \sec x$; (b) $\psi = A \tan x$; (c) $\psi = A \exp(x^2)$; (d) $\psi = A \exp(-x^2)$.

2 The wave function of a certain particle is $\psi = A \cos^2 x$ for $-\pi/2 < x < \pi/2$. (a) Find the value of A. (b) Find the probability that the particle be found between $x = 0$ and $x = \pi/4$.

3 Show that the expectation values <px> and <xp> are related by $< px > - < xp > = \hbar / i$

4 At time $t = 0$ a particle is represented by the wave function

$$\psi(x,0) = \begin{cases} A\dfrac{x}{a}; & \text{if } 0 \le x \le a \\ A\dfrac{(b-x)}{(b-a)}; & \text{if } a \le x \le b \\ 0; & \text{otherwise} \end{cases}$$

where, A, a and b are constants.

(a) Normalize ψ (i.e., determine A)
(b) Sketch $\psi(x,0)$, as a function of x
(c) Where is the particle most likely to be found at $t = 0$?
(d) What is the probability of finding the particle to the left of a? Check your result in the limiting cases $b = a$ and $b = 2a$.
(e) What is the expectation value of x? [Note: The expectation value $< x >$ is defined as $< x >= \int_{\infty}^{\infty} x \mid \psi \mid^2 dx$, where ψ is a normalized wave function.].