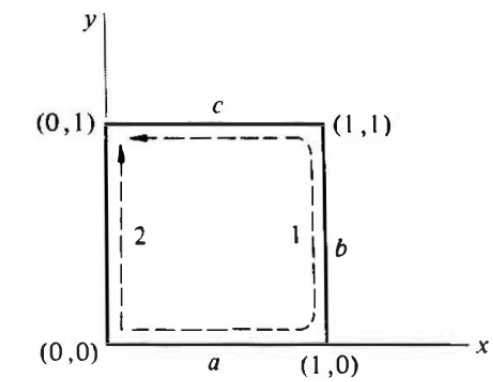
INDIAN INSTITUTE OF TECHNOLOGY PATNA

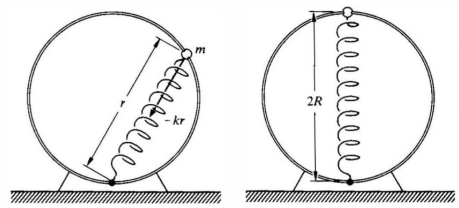
PH103 : Physics
Tutorial 4

1. Let $\vec{F} = A(xy\hat{i} + y^2\hat{j})$, and consider the integral from $(0, 0)$ to $(0, 1)$, first along path 1 and then along path 2, as shown in the figure. Find the work done in moving the particle along path 1 and path 2.



2. A bead of mass m slide without friction on a vertical hoop of radius R. The bead moves under the combined action of gravity and a spring attached to the bottom of the hoop. For simplicity, we assume that the equilibrium length of the spring is zero, so that the force due to the spring is $-kr$, where r is the instantaneous length of the spring as shown in the figure.

   The bead is released at the top of the hoop with negligible speed. How fast is the bead moving at the bottom of the hoop ?

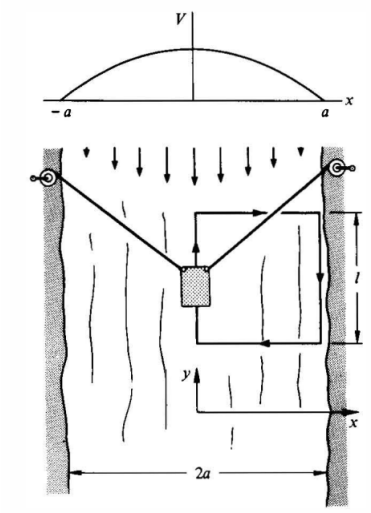


3. We know that the gravitational force is conservative since it possesses a potential energy function. Prove that the force of gravity is conservative by showing that its curl is zero.

4. Consider a river with a current whose velocity $\vec{V}$ is the maximum at the center and drops to zero at either bank, represented by

$$\vec{V} = -V_0\left(1 - \frac{x^2}{a^2}\right)\hat{j}$$

   The width of the river is $2a$, and the coordinates are shown in the sketch. Suppose that a barge in the stream is hauled around the path shown, by winches on the banks. Find the work done by the winches in pulling the barge around the path shown.



5. What is the force associated with $V = -\frac{A}{3}r^2 \cos\theta$ in the spherical polar co-ordinate?

6. Evaluate $\int\int\int 4xydV$ inside the volume bounded by $z = 2x^2 + 2y^2 - 7$ and $z = 1$.