# INDIAN INSTITUTE OF TECHNOLOGY PATNA

## PH103 : Physics Tutorial 3

1. A vector field is given by $\vec{A} = x^2\hat{e}_x + y^2\hat{e}_y + z^2\hat{e}_z$. Determine the surface integral $\int\int \vec{A}.\vec{ds}$ over the closed surface of a cylinder $x^2 + y^2 = 16$ bounded by the planes z=0, z=8.

2. Determine the change of the function $df(x, y, z)$, given $f(x, y, z) = \sqrt{(x^2 + y^2)}$ in the dierction $\hat{e}_x + \hat{e}_y + \hat{e}_z$ at the point (2,0,1)

3. Using cylindrical polar co-ordinate system, Show that $\vec{\nabla}.\vec{r} = 3$ and $\vec{\nabla} \times \vec{r} = 0$

4. A particle is taken along the path $A(0, 0, 0) \rightarrow B(d_1, 0, 0) \rightarrow C(d_1, d_2, 0) \rightarrow D(0, d_2, 0) \rightarrow A(0, 0, 0)$ against a force field given by $F(x, y, z) = 4xy\hat{e_x} - 2x^2\hat{e_y} + 3z\hat{e_z}$, where the various constants have appropriate dimensions. Determine the work done by this force, and verify the Stokes theorem by finding the work done using both the line integral and the surface integral that appears in it. The stokes theorem is given as $\oint \vec{F}.\vec{dl} = \int\int(\vec{\nabla} \times \vec{F}).\vec{ds}$

5. A rigid body is rotating about a fixed axis with a constant angular velocity $\vec{\omega}$. Take $\vec{\omega}$ to lie along the z-axis. Express the position vector $\vec{r}$ in cylindrical polar co-ordinates and by using cylindrical polar co-ordinates, calculate $\vec{v} = \vec{\omega} \times \vec{r}$. Also calculate $\vec{\nabla} \times \vec{v}$.

6. A force is described by

   $\vec{F} = -\hat{e}_x \frac{y}{x^2+y^2} + \hat{e}_y \frac{x}{x^2+y^2}$

   (a) Express $\vec{F}$ in cylindrical polar co-ordinates
   (b) Calculate curl of $\vec{F}$ in cylindrical polar co-ordinates
   (c) Calculate the work done by $\vec{F}$ in encircling the unit circle once counter-clockwise. How do you reconcile the results of (b) and (c)?